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AUXILIARY ESTIMATING FUNCTIONS FOR DOUBLY
TRUNCATED NORMAL SAMPLES

by J. DAVID LIFSEY
Aero-Astrodynamics Laboratory

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J. David Lifsey

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ABSTRACT

When sampling procedures on a random variable X are such that the resulting sample consists of N measured observations for which $X_\alpha < X < X_\beta$, and no information is known for $X < X_\alpha$ and $X > X_\beta$, the sample is said to be doubly truncated at the known terminals X_α and X_β . To calculate maximum likelihood estimates of the mean and standard deviation of a normally distributed population from doubly truncated samples, it is necessary to solve simultaneously a pair of rather complex nonlinear estimating equations. Since every estimate is a function of the sample values and must be regarded as an observed value of a certain random variable, there are no means of predicting in a given case, the true population value assumed by the estimate. The "goodness" of an estimate cannot be judged from individual values, but only from the distribution of the values which it will assume in the long run, i.e., from its sampling distribution. Some estimate of the variance of these sample estimates is needed. Values of auxiliary functions required to obtain, from doubly truncated normal samples, maximum likelihood estimates of parameters of the parent population and the asymptotic (large sample) variances and covariance of these estimates are given.

NASA - GEORGE C. MARSHALL SPACE FLIGHT CENTER

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AUXILIARY ESTIMATING FUNCTIONS FOR
DOUBLY TRUNCATED NORMAL SAMPLES

By

J. David Lifsey

TERRESTRIAL ENVIRONMENT GROUP
AEROSPACE ENVIRONMENT OFFICE
AERO-ASTRODYNAMICS LABORATORY

TECHNICAL MEMORANDUM X-53221

AUXILIARY ESTIMATING FUNCTIONS FOR
DOUBLY TRUNCATED NORMAL SAMPLES

SUMMARY

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Restricted samples are those selected under conditions such that full observation is not permitted over some portion of the population range of values. The problem of estimating population parameters from samples obtained under such conditions is frequently encountered in scientific investigations, with restricted samples arising quite naturally in such fields as life testing, dosage and mortality response studies, biological assays, industrial production of some measurable item, quality control, and many areas of engineering. When a sample of a random variable X results in N measured observations for which $X_\alpha < X < X_\beta$, with no information known about $X < X_\alpha$ or $X > X_\beta$, the sample is described as being doubly truncated at the points X_α and X_β . To calculate maximum likelihood estimates for the mean and standard deviation of a normally distributed population from doubly truncated normal samples, it is necessary to solve simultaneously a pair of rather complex nonlinear estimating equations. In 1957, Cohen [2] gave methods employing a graph of estimation curves for reducing the computational labor required to obtain these solutions. In this paper, a graph is given and auxiliary functions are evaluated to further aid in obtaining solutions to the estimating equations. The asymptotic (large sample) variances and covariance of these estimates are also evaluated.

I. INTRODUCTION

Restricted samples are those selected under conditions such that full observation is not permitted over some portion of the population range of values. Restricted sampling was dealt with in statistics as early as 1807 when Sir Francis Galton [4] encountered a singly truncated sample in studying the distribution of the time taken by trotting horses to run a measured course. Galton was only able to sample from the times of the horses that had qualified, since the times of horses failing to qualify were not recorded. The resulting samples were thus singly truncated on the right at the known terminus of $t = 150$ seconds, since to qualify it was necessary to better this time over a one-mile course.

Galton assumed his distributions to be normal and used sample modes as estimates of the population means. He simply plotted frequency polygons and located the required values by inspection. With modes equated to medians he located sample quartiles and used semi-interquartile ranges to estimate population standard deviations.

In 1902 Karl Pearson [9] recalculated Galton's estimates using a procedure for estimating parameters of a normal population by fitting least-square parabolas to logarithms of the truncated sample frequencies. Karl Pearson and Alice Lee [10] in 1908 used the method of moments [7] to obtain estimates of the mean and standard deviation for a singly truncated sample and gave tables to aid in the computation of these estimates.

Further publications on this problem did not appear until 1931, when R. A. Fisher [3] gave results obtained using the method of maximum likelihood [7], which he had introduced ten years earlier. Fisher considered samples of the same type as those studied by Pearson and Lee and demonstrated that, for the singly truncated normal case, the method of maximum likelihood gives estimates identical with those obtained by the method of moments. He also gave asymptotic variances and covariances of the maximum likelihood estimates.

The terms singly truncated, doubly truncated, singly censored, etc., used in connection with restricted sampling were not used by the early writers but were introduced rather slowly into the literature. Stevens [11] in 1937 was the first to consider the doubly censored sample, although he did not use this term. He published maximum likelihood estimating equations for normal population samples of the type which are now designated as singly and doubly censored samples with known terminals.

Additional results on this topic have been published by numerous authors including Hald, Birnbaum, Halperin, Gupta, Sampford, Des Raj, Bliss, Cohen and others. In 1949 Cohen [1] derived formulas whereby certain special functions required in solution of the problem as given by Pearson and Lee and by Fisher might be readily evaluated using only an ordinary table of areas and ordinates of the normal curve. Thus, it became possible to obtain the desired estimates with an improvement in accuracy whether or not the special tables required by the other two methods were available.

To estimate population parameters from restricted samples, the techniques most widely used are the method of moments, the method of maximum likelihood, and order statistics. Fisher [3] observed that, for singly truncated normal samples, the method of moments and the method of maximum likelihood yield identical estimates for the parameters. Hotelling [5] demonstrated that the two methods also lead to identical estimates in the case of truncated samples from multinormal

distributions. Subsequently, Tukey [14] proved that, if any family of distributions admits a set of sufficient statistics [7], the family obtained by truncation to a fixed set or by fixed selection also admits the same set of sufficient statistics. Since the mean and variance are sufficient statistics for a multinormal distribution, the above results guarantee that the method of moments and the method of maximum likelihood lead to identical estimates of these parameters from doubly truncated normal samples. In this paper, the mean and standard deviation of a normally distributed population are estimated from doubly truncated normal samples by the method of maximum likelihood. This choice was governed by the above properties and by the fact that the method of order statistics seemed to offer no particular advantage. Further, similar results have been obtained and are available to check these results.

Cohen [2] gave graphical aids for obtaining solutions to maximum likelihood estimating equations for the mean and standard deviation of doubly truncated normal samples; this writer [6] modified Cohen's results and also gave tabular values of asymptotic (large sample) variance and covariance factors for these estimates. The latter work is extended in this report.

II. MAXIMUM LIKELIHOOD ESTIMATING EQUATIONS

The probability density function of a random variable x taken from a normally distributed population is given by

$$f(x) = (\sigma\sqrt{2\pi})^{-1} \exp [-(x - \mu)^2/2\sigma^2], \quad -\infty \leq x \leq \infty. \quad (1)$$

Let x_0 be a known fixed value of the random variable x and designate x_0 as the left terminus or truncation point. Let $x_0 + w$ be another known fixed value of the random variable x and designate it as the right terminus or truncation point; w is the range of truncation. Let n be the number of measured observations such that $x_0 < x < x_0 + w$. In population standard units, the left and right truncation points become, respectively,

$$\xi_1 = (x_0 - \mu)/\sigma \quad \text{and} \quad \xi_2 = (x_0 + w - \mu)/\sigma = \xi_1 + w/\sigma. \quad (2)$$

If $F(\xi)$ denotes the distribution function of ξ , the probability that a selected value of the random variable ξ has the requirements for inclusion in a sample that is doubly truncated at ξ_1 and ξ_2 becomes

$$P(\xi_1 \leq \xi \leq \xi_2) = F(\xi_2) - F(\xi_1), \quad (3)$$

where

$$F(\xi_i) = \int_{-\infty}^{\xi_i} \varphi(t) dt, \quad i = 1, 2$$

and

$$\varphi(t) = (\sqrt{2\pi})^{-1} \exp(-t^2/2), \quad t = (x - \mu)/\sigma,$$

the standardized normal variate. Equation (3) represents the area under the normal curve between ordinates erected at ξ_1 and ξ_2 . The probability density function for this case can be written as

$$f(x) = \left[\sigma \sqrt{2\pi} (F_2 - F_1) \right]^{-1} \exp \left[-(x - \mu)^2 / 2\sigma^2 \right], \quad x_0 \leq x \leq x_0 + w, \quad (4)$$

where $F_i = F(\xi_i)$, $i = 1, 2$. The likelihood function for (4) is

$$P = L(x_1, x_2, \dots, x_n; \mu, \sigma) = \left[\sigma \sqrt{2\pi} (F_2 - F_1) \right]^{-n} \exp \left[- \sum_{i=1}^n (x_i - \mu)^2 / 2\sigma^2 \right]. \quad (5)$$

Taking the logarithm of (5) for ease of differentiation gives

$$L = \ln P = \text{const.} - n \ln \sigma - n \ln (F_2 - F_1) - \left[\sum_{i=1}^n (x_i - \mu)^2 / 2\sigma^2 \right]. \quad (6)$$

For the likelihood function to be a maximum, it is necessary that

$$\frac{\partial L}{\partial \mu} = 0 = \frac{\partial L}{\partial \sigma}.$$

Taking the required first partials and equating to zero yields

$$\frac{\partial L}{\partial \mu} = -n(Z_1 - Z_2)/\sigma + \sum_{i=1}^n (x_i - \mu)/\sigma^2 = 0, \quad (7)$$

$$\frac{\partial L}{\partial \sigma} = -n/\sigma - n(\xi_1 Z_1 - \xi_2 Z_2)/\sigma + \sum_{i=1}^n (x_i - \mu)^2 / \sigma^3 = 0, \quad (8)$$

where

$$Z_i = Z(\xi_i) = \varphi(\xi_i)/(F_2 - F_1), \quad i = 1, 2.$$

Let

$$m_k = \sum_{i=1}^n (x_i - x_o)^k / n$$

designate the k^{th} sample moment about x_o . From (2)

$$\sigma = w / (\xi_2 - \xi_1) \quad \text{and} \quad \mu = x_o - \sigma \xi_1. \quad (9)$$

Equations (7) and (8) may be written as

$$\sum_{i=1}^n (x_i - \mu)/n = \sigma(z_1 - z_2) \quad (10)$$

and

$$\sum_{i=1}^n (x_i - \mu)^2/n = \sigma^2(1 + \xi_1 z_1 - \xi_2 z_2). \quad (11)$$

Expanding the left-hand side of (10) and substituting from (9) results in

$$\bar{x} - x_0 + \sigma \xi_1 = \sigma(z_1 - z_2), \quad (12)$$

and consequently

$$(z_1 - z_2 - \xi_1)/(\xi_2 - \xi_1) - m_1/w = 0, \quad (13)$$

which is the first estimating equation. Squaring (10) and subtracting the result from (11) gives

$$\sum_{i=1}^n (x_i - \mu)^2/n - \left[\sum_{i=1}^n (x_i - \mu)/n \right]^2 = \sigma^2 \left[1 + \xi_1 z_1 - \xi_2 z_2 - (z_1 - z_2)^2 \right]. \quad (14)$$

Expansion and simplification of the left-hand side of (14) yields

$$\sum_{i=1}^n x_i^2/n - \bar{x}^2 = s^2,$$

the sample variance. From (9),

$$\sigma^2 = w^2 / (\xi_2 - \xi_1)^2.$$

Hence, equation (14) reduces to

$$[1 + \xi_1 z_1 - \xi_2 z_2 - (z_1 - z_2)^2] / (\xi_2 - \xi_1)^2 - s^2/w^2 = 0, \quad (15)$$

which is the second estimating equation.

For any given sample of size n , the quantities m_1/w ($m_1 = \bar{x} - x_0$) and s^2/w^2 may be computed and the estimating equations

$$(z_1 - z_2 - \hat{\xi}_1) / (\hat{\xi}_2 - \hat{\xi}_1) - m_1/w = 0 \quad (16)$$

$$[1 + \hat{\xi}_1 z_1 - \hat{\xi}_2 z_2 - (z_1 - z_2)^2] / (\hat{\xi}_2 - \hat{\xi}_1)^2 - s^2/w^2 = 0 \quad (17)$$

solved simultaneously for $\hat{\xi}_1$ and $\hat{\xi}_2$. Consequently, with these values determined, $\hat{\sigma}$ and $\hat{\mu}$ follow from (9) as

$$\begin{aligned} \hat{\sigma} &= w / (\hat{\xi}_2 - \hat{\xi}_1) \\ \hat{\mu} &= x_0 - \hat{\sigma} \hat{\xi}_1 \end{aligned} \quad (18)$$

(The caret (^) serves to distinguish maximum likelihood estimators or estimates from the parameters being estimated.) Except for slight changes in notation, equations (16) and (17) correspond to those given by Cohen [2].

III. SOLUTION OF THE ESTIMATING EQUATIONS

The simultaneous solution of equations (16) and (17) is often a laborious task since they are nonlinear and neither equation can be expressed explicitly as a function of the other. Hence, their solution may require some type of iterative procedure or Newton-Raphson method [15]. Employing the notation

$$H_1(\xi_1, \xi_2) = (Z_1 - Z_2 - \xi_1)/(\xi_2 - \xi_1), \quad (19)$$

$$H_2(\xi_1, \xi_2) = [1 + \xi_1 Z_1 - \xi_2 Z_2 - (Z_1 - Z_2)^2]/(\xi_2 - \xi_1)^2, \quad (20)$$

estimating equations (16) and (17) have the form

$$F_1(\xi_1, \xi_2) = H_1(\xi_1, \xi_2) - k_1 = 0, \quad (21)$$

$$F_2(\xi_1, \xi_2) = H_2(\xi_1, \xi_2) - k_2 = 0, \quad (22)$$

and, hence, represent two families of curves for various values of k_1 and k_2 . For a given sample, k_1 and k_2 are constants. If the substitution $\xi_1 = -\xi_2$ and $\xi_2 = -\xi_1$ is made, (18) becomes

$$H_1(-\xi_2, -\xi_1) = - (Z_1 - Z_2 - \xi_2)/(\xi_2 - \xi_1).$$

Adding and subtracting ξ_1 in the numerator and making an algebraic simplification gives

$$H_1(-\xi_2, -\xi_1) = 1 - [(Z_1 - Z_2 - \xi_1)/(\xi_2 - \xi_1)] = 1 - H_1(\xi_1, \xi_2), \quad (23)$$

which shows the graph of $H_1(\xi_1, \xi_2) - k = 0$ to be a reflection of the graph of $H_1(\xi_1, \xi_2) - (1 - k) = 0$ about the line $\xi_1 + \xi_2 = 0$. Thus, in plotting $H_1(\xi_1, \xi_2)$, one-half the points may be obtained by reflection.

Let $\xi_1 = -\xi_2$ and $\xi_2 = -\xi_1$ in (20). Then,

$$H_2(-\xi_2, -\xi_1) = [1 - \xi_2 Z_2 + \xi_1 Z_1 - (Z_2 - Z_1)^2] / (-\xi_1 + \xi_2)^2 = H_2(\xi_1, \xi_2), \quad (24)$$

which shows the graph of $H_2(\xi_1, \xi_2) - k = 0$ to be symmetric with respect to the graph of $H_2(-\xi_2, -\xi_1) - (1 - k) = 0$ about the line $\xi_1 + \xi_2 = 0$. In plotting $H_2(\xi_1, \xi_2)$, one-half the points may be obtained by symmetry.

As an aid to solving (16) and (17), Cohen [2] graphed the families of curves given by (21) and (22). His procedure was to compute $H_1 = (\bar{x} - x_0)/w$ and $H_2 = s^2/w^2$ from sample data and then read $\hat{\xi}_1$ and $\hat{\xi}_2$ as coordinates of the intersection point of the two curves. These initial estimates served as first approximations for subsequent improvement by an iterative procedure. Cohen's values of $H_1(\xi_1, \xi_2)$ and $H_2(\xi_1, \xi_2)$ were obtained from abbreviated tables compiled by Thompson, Friedman and Garelis [13] who tabulated their values at intervals of 0.5 for the arguments ξ_1 and ξ_2 . An extension of these tables [6] was used to plot curves of $\xi_1 = \text{const.}$ against the arguments $H_1(\xi_1, \xi_2)$ and $H_2(\xi_1, \xi_2)$, the inverse of the presentation used by Cohen [2]. An improvement in accuracy of the initial estimates was obtained by this procedure.

In an effort to further improve the accuracy of the initial estimates of solutions to the estimating equations, new values of $H_1(\xi_1, \xi_2)$ and $H_2(\xi_1, \xi_2)$ were computed and rounded to 10D for the arguments

$$\xi_1 = -5.0(0.1) -0.1$$

and

$$\xi_2 = \xi_1 + 0.5(0.1) -\xi_1.$$

The computations were performed on an IBM 1620 computer using input values (to 15D) of the normal curve ordinates and area [9]. Functional values of $Z_1(\xi_1, \xi_2)$ and $Z_2(\xi_1, \xi_2)$ were also computed and, together with $H_1(\xi_1, \xi_2)$ and $H_2(\xi_1, \xi_2)$ values, rounded to 10D prior to computer output on punched cards, which were machine listed for inclusion in Table I. For the family of curves $\xi_1 = \text{const.}$, the tabulated values of $H_1(\xi_1, \xi_2)$ and $H_2(\xi_1, \xi_2)$ were plotted and appear as Figure 1, which may be used to

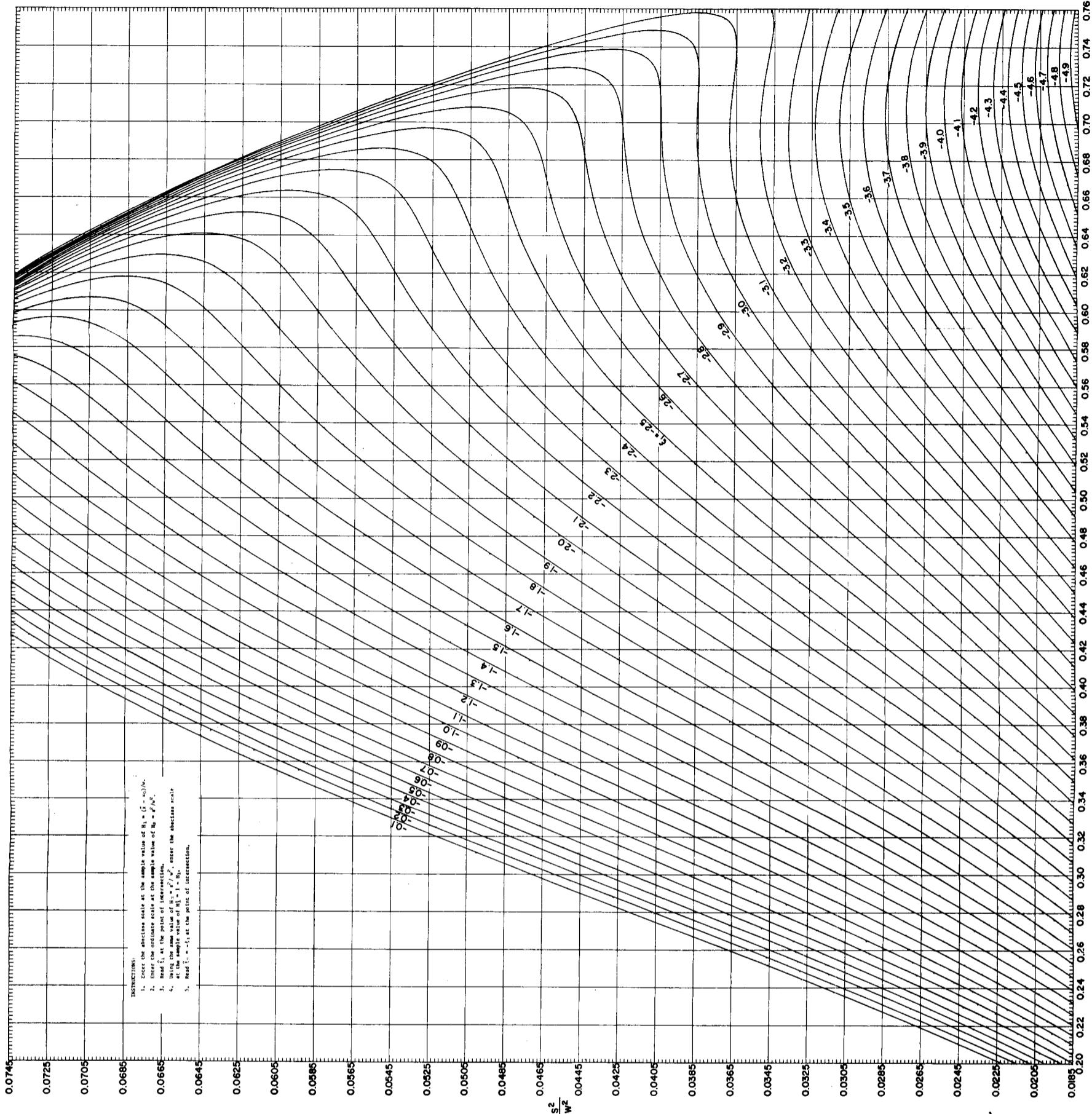


FIG. I. ESTIMATION CURVES FOR DOUBLY TRUNCATED NORMAL SAMPLES

obtain initial values of the estimates $\hat{\xi}_1$ and $\hat{\xi}_2$. Sample values of $H_1 = (\bar{x} - x_0)/w$ and $H_2 = s^2/w^2$ are computed and used to enter Figure 1. $\hat{\xi}_1$ is read at the intersection point of these values, and $\hat{\xi}_2 = -\hat{\xi}_1$ is read at the intersection point of $H'_1 = 1 - H_1$ and H_2 . The values of $\hat{\xi}_1$ and $\hat{\xi}_2$ obtained in this manner are then used to solve the estimating equations (18) or, if greater accuracy is required, they may be used as first approximations to be improved upon by an iterative procedure.

IV. VARIANCE OF THE ESTIMATES

The asymptotic (large sample) variance-covariance matrix of the estimates $\hat{\mu}$ and $\hat{\sigma}$ obtained from (18), is found by inverting the matrix whose elements are negatives of the expected value of the second order partial derivatives of logarithms of the likelihood function (6), i.e., by inverting the information matrix

$$I(\mu, \sigma) = \begin{pmatrix} -E\left[\frac{\partial^2 L}{\partial \mu^2}\right] & -E\left[\frac{\partial^2 L}{\partial \mu \partial \sigma}\right] \\ -E\left[\frac{\partial^2 L}{\partial \mu \partial \sigma}\right] & -E\left[\frac{\partial^2 L}{\partial \sigma^2}\right] \end{pmatrix}, \quad (25)$$

where $E[]$ denotes the expected value of the quantity in brackets. Since,

$$\begin{aligned} \frac{\partial \xi_i}{\partial \mu} &= -\frac{1}{\sigma}, & \frac{\partial \xi_i}{\partial \sigma} &= -\frac{\xi_i}{\sigma}, \\ \frac{\partial F_i}{\partial \mu} &= -\frac{\varphi_i}{\sigma}, & \frac{\partial F_i}{\partial \sigma} &= -\frac{\xi_i \varphi_i}{\sigma}, \\ \frac{\partial Z_i}{\partial \mu} &= \frac{\xi_i z_i - z_i(z_1 - z_2)}{\sigma}, & \frac{\partial Z_i}{\partial \sigma} &= \frac{\xi_i^2 z_i - z_i(\xi_1 z_1 - \xi_2 z_2)}{\sigma}, \end{aligned}$$

(i = 1, 2)

it follows from (7) and (8) that

$$\frac{\partial^2 L}{\partial \mu^2} = \frac{-n}{\sigma^2} [1 + \xi_1 z_1 - \xi_2 z_2 - (z_1 - z_2)^2], \quad (26)$$

$$\frac{\partial^2 L}{\partial \sigma^2} = \frac{-n}{\sigma^2} [-(1 + \xi_1 z_1 - \xi_2 z_2)^2 + \xi_1^3 z_1 - \xi_2^3 z_2] - \frac{3}{\sigma^2} \sum_{i=1}^n (x_i - \mu)^2 / \sigma^2, \quad (27)$$

$$\frac{\partial^2 L}{\partial \mu \partial \sigma} = \frac{-n}{\sigma^2} [-(z_1 - z_2)(1 - \xi_1 z_1 - \xi_2 z_2) + \xi_1^2 z_1 - \xi_2^2 z_2] - \frac{2}{\sigma^2} \sum_{i=1}^n (x_i - \mu) / \sigma. \quad (28)$$

Inspection reveals that the only expected values needed are $E[(x_i - \mu)^2 / \sigma^2]$ in (27) and $E[(x_i - \mu) / \sigma]$ in (28), since these are the only quantities which contain the variate x . It can be shown that

$$E[(x_i - \mu)^2 / \sigma^2] = 1 + \xi_1 z_1 - \xi_2 z_2$$

and

$$E[(x_i - \mu) / \sigma] = z_1 - z_2.$$

From these results, the required negatives of the expected values of the second order partial derivatives of the likelihood function (6) are obtained as

$$\left. \begin{aligned} -E\left[\frac{\partial^2 L}{\partial \mu^2}\right] &= \frac{n}{\hat{\sigma}^2} \hat{\phi}_{11} \\ -E\left[\frac{\partial^2 L}{\partial \sigma^2}\right] &= \frac{n}{\hat{\sigma}^2} \hat{\phi}_{22} \\ -E\left[\frac{\partial^2 L}{\partial \mu \partial \sigma}\right] &= \frac{n}{\hat{\sigma}^2} \hat{\phi}_{12} \end{aligned} \right\}, \quad (29)$$

where

$$\hat{\phi}_{ij} = \varphi_{ij}(\hat{\xi}_1, \hat{\xi}_2), \quad i, j = 1, 2$$

and

$$\left. \begin{aligned} \hat{\phi}_{11} &= 1 + \hat{\xi}_1 z_1 - \hat{\xi}_2 z_2 - (z_1 - z_2)^2 \\ \hat{\phi}_{12} &= (z_1 - z_2)[1 - (\hat{\xi}_1 z_1 - \hat{\xi}_2 z_2)] + \hat{\xi}_1^2 z_1 - \hat{\xi}_2 z_2 \\ \hat{\phi}_{22} &= 2 + (\hat{\xi}_1 z_1 - \hat{\xi}_2 z_2)[1 - (\hat{\xi}_1 z_1 - \hat{\xi}_2 z_2)] + \hat{\xi}_1^3 z_1 - \hat{\xi}_2^3 z_2 \end{aligned} \right\}. \quad (30)$$

Substitution in (25) gives

$$I(\hat{\mu}, \hat{\sigma}) = \begin{pmatrix} \frac{n}{\hat{\sigma}^2} \hat{\phi}_{11} & \frac{n}{\hat{\sigma}^2} \hat{\phi}_{12} \\ \frac{n}{\hat{\sigma}^2} \hat{\phi}_{12} & \frac{n}{\hat{\sigma}^2} \hat{\phi}_{22} \end{pmatrix}. \quad (31)$$

The asymptotic (large sample) variance-covariance matrix is found by inverting (31). Hence,

$$V(\hat{\mu}, \hat{\sigma}) = I^{-1}(\hat{\mu}, \hat{\sigma}) = \begin{pmatrix} \frac{\hat{\sigma}^2}{n} \hat{\mu}_{11} & \frac{\hat{\sigma}^2}{n} \hat{\mu}_{12} \\ \frac{\hat{\sigma}^2}{n} \hat{\mu}_{12} & \frac{\hat{\sigma}^2}{n} \hat{\mu}_{22} \end{pmatrix}, \quad (32)$$

with

$$\left. \begin{aligned} \text{Var}(\hat{\mu}) &\sim (\hat{\sigma}^2/n) \hat{\mu}_{11} \\ \text{Var}(\hat{\sigma}) &\sim (\hat{\sigma}^2/n) \hat{\mu}_{22} \\ \text{Cov}(\hat{\mu}, \hat{\sigma}) &\sim (\hat{\sigma}^2/n) \hat{\mu}_{12} \end{aligned} \right\} \quad (33)$$

and

$$\hat{\mu}_{ij} = \mu_{ij}(\hat{\xi}_1, \hat{\xi}_2), \quad i, j = 1, 2,$$

where

$$\left. \begin{aligned} \hat{\mu}_{11} &= \hat{\phi}_{22}/(\hat{\phi}_{11}\hat{\phi}_{22} - \hat{\phi}_{12}^2) \\ \hat{\mu}_{12} &= -\hat{\phi}_{12}/(\hat{\phi}_{11}\hat{\phi}_{22} - \hat{\phi}_{12}^2) \\ \hat{\mu}_{22} &= \hat{\phi}_{11}/(\hat{\phi}_{11}\hat{\phi}_{22} - \hat{\phi}_{12}^2) \end{aligned} \right\}. \quad (34)$$

The coefficient of correlation between the variance estimates may be expressed as

$$\rho(\hat{\mu}, \hat{\sigma}) = \hat{\mu}_{12}/\sqrt{\hat{\mu}_{11}\hat{\mu}_{22}}. \quad (35)$$

If in (30) the substitution $\hat{\xi}_1 = -\hat{\xi}_2$ and $\hat{\xi}_2 = -\hat{\xi}_1$ is made, it follows that

$$\left. \begin{aligned} \mu_{11}(-\hat{\xi}_2, -\hat{\xi}_1) &= \mu_{11}(\hat{\xi}_1, \hat{\xi}_2) \\ \mu_{12}(-\hat{\xi}_2, -\hat{\xi}_1) &= -\mu_{12}(\hat{\xi}_1, \hat{\xi}_2) \\ \mu_{22}(-\hat{\xi}_2, -\hat{\xi}_1) &= \mu_{22}(\hat{\xi}_1, \hat{\xi}_2) \\ \rho(-\hat{\xi}_2, -\hat{\xi}_1) &= -\rho(\hat{\xi}_1, \hat{\xi}_2) \end{aligned} \right\}. \quad (36)$$

To simplify calculation of the asymptotic (large sample) variances and covariance of the estimates $\hat{\mu}$ and $\hat{\sigma}$, the auxiliary functions $\mu_{ij}(\xi_1, \xi_2)$ $i, j = 1, 2$, and $\rho(\xi_1, \xi_2)$ were evaluated on the IBM 1620 computer and rounded to 10D for the same arguments used to evaluate the functions of Section III. Likewise, these values were machine listed from punched cards and are included in Table I. With the exception of one earlier but less extensive table [6], previous tabulation of these variance factors for doubly truncated normal samples has not been seen by this writer.

For any doubly truncated sample, after reading ξ_1 and ξ_2 from Figure 1, enter the appropriate columns of Table I and interpolate to obtain the required values of $\mu_{ij}(\xi_1, \xi_2)$ $i, j = 1, 2$. The asymptotic variances and covariance may then be approximated with (33), the estimate of $\hat{\sigma}^2$, and the sample size n . Similarly, $\rho(\hat{\mu}, \hat{\sigma})$ may be approximated from (35).

V. EFFICIENCY OF THE ESTIMATES

The question of joint efficiency of the estimates $\hat{\mu}$ and $\hat{\sigma}$ arises naturally at this point. Swamy [12] demonstrates that the joint efficiency of BAN (best asymptotically normal) estimate [7] of μ and σ based on incomplete (singly or doubly truncated) samples of size n depends on the points of truncation ξ_1 and ξ_2 , monotonically increasing to the joint efficiency of BAN estimates of μ and σ based on complete samples of the same size as the distance between the points of truncation increases. Since maximum likelihood estimates are also BAN estimates, it follows that the maximum likelihood estimates of μ and σ based on a doubly truncated sample are jointly less efficient than those based on the complete sample for fixed n .

The joint efficiency of the estimates $\hat{\mu}$ and $\hat{\sigma}$ based on doubly truncated samples is defined as .

$$e_{DT}(\xi_1, \xi_2) = \frac{|I_{DT}(\mu, \sigma)|}{|I(\mu, \sigma)|} ,$$

which is the ratio of the determinantal value of the information matrix (25) for a doubly truncated sample of size n to the determinantal value of this matrix for a complete sample of size n . In terms of the μ_{ij} notation of Section IV, efficiency may be expressed as

$$e_{DT}(\xi_1, \xi_2) = \frac{|M(\xi_1, \xi_2)|}{|M_{DT}(\xi_1, \xi_2)|} , \quad (37)$$

where

$$M(\xi_1, \xi_2) = \begin{pmatrix} \mu_{11}(\xi_1, \xi_2) & \mu_{12}(\xi_1, \xi_2) \\ \mu_{12}(\xi_1, \xi_2) & \mu_{22}(\xi_1, \xi_2) \end{pmatrix}.$$

For a complete sample of size n ,

$$M(\xi_1, \xi_2) = \begin{pmatrix} 1 & 0 \\ 0 & 0.5 \end{pmatrix} \quad \text{and} \quad |M(\xi_1, \xi_2)| = 0.5.$$

Hence,

$$e_{DT}(\xi_1, \xi_2) = \frac{0.5}{|M_{DT}(\xi_1, \xi_2)|} = \frac{0.5}{\mu_{11}\mu_{22} - \mu_{12}^2} \quad (38)$$

and this quantity may be evaluated directly from Table I.

VI. ILLUSTRATIVE EXAMPLE

To illustrate estimation of μ and σ from a doubly truncated normal sample, the methods developed here will be applied to the example given by Cohen [2].

The entire production of a certain bushing is sorted through go, no-go gauges, with the result that items of diameter in excess of 0.6015 inches and those less than 0.5985 inches are discarded. A random sample of 75 bushings selected from the screened population resulted in

$$\bar{x} = 6.0014933 \times 10^{-1}, \quad s^2 = 3.71187 \times 10^{-7},$$

$$x_0 = 5.985 \times 10^{-1}, \quad w = 3.0 \times 10^{-3}.$$

Thus,

$$H_1 = (\bar{x} - x_0)/w = 5.49777 \times 10^{-1},$$

$$H'_1 = 1 - H_1 = 4.5022 \times 10^{-1},$$

$$H_2 = s^2/w^2 = 4.1243 \times 10^{-2}.$$

Interpolation in Figure 1 gives

$$\hat{\xi}_1 = -2.525 \quad \text{and} \quad \hat{\xi}_2 = 2.000$$

as estimates of ξ_1 and ξ_2 . Substituting in (18) yields

$$\hat{\sigma} = w / (\hat{\xi}_2 - \hat{\xi}_1) = 0.0030 / 4.525 = 6.62983 \times 10^{-4}$$

$$\hat{\mu} = x_0 - \hat{\sigma}\hat{\xi}_1 = 0.5985 - (6.62983 \times 10^{-4})(-2.525) = 6.0017403 \times 10^{-1}.$$

From Table I, variance and covariance factors and the correlation between the estimates are interpolated as

$$\mu_{11}(\hat{\xi}_1, \hat{\xi}_2) = 1.21710, \quad \mu_{12}(\hat{\xi}_1, \hat{\xi}_2) = 0.17458,$$

$$\mu_{22}(\hat{\xi}_1, \hat{\xi}_2) = 0.93069, \quad \rho(\hat{\mu}, \hat{\sigma}) = 0.16411.$$

From (38), the joint efficiency of the estimates $\hat{\mu}$ and $\hat{\sigma}$ is

$$e_{DT}(-2.525, 2.000) = 0.5 / 1.10226 = 0.4536.$$

With $\hat{\sigma}^2 = 4.39546 \times 10^{-7}$ and $\mu = 75$, the variances and covariance of the estimates $\hat{\mu}$ and $\hat{\sigma}$ are approximated by (33) as

$$V(\hat{\mu}) \sim 7.13295 \times 10^{-9}, \quad V(\hat{\sigma}) \sim 5.45441 \times 10^{-9},$$

$$\text{Cov } (\hat{\mu}, \hat{\sigma}) \sim 1.02315 \times 10^{-9}.$$

Standard errors of the estimates follow as

$$\sigma(\hat{\mu}) = \sqrt{V(\hat{\mu})} \sim 8.44568 \times 10^{-5},$$

$$\sigma(\hat{\sigma}) = \sqrt{V(\hat{\sigma})} \sim 7.38540 \times 10^{-5}.$$

VII. CONCLUSIONS

The auxiliary estimating functions given in equations (19) and (20) considerably reduce the computational effort required to estimate the parameters μ and σ of a normal population from doubly truncated samples. Figure 1 and Table I should prove adequate for obtaining these maximum likelihood estimates in most practical situations. Reliability of these estimates can be obtained from equation (33) and the auxiliary estimating functions (34) and (35), also tabulated in Table I. The joint efficiency of $\hat{\mu}$ and $\hat{\sigma}$ follows from (38).

To evaluate the auxiliary functions (19), (20), (34) and (35) only areas and ordinates of the normal curve are required. Such tables [8] are readily available; however, if computer subroutines are available to calculate the normal areas and ordinates, evaluation of the auxiliary functions could be reduced to simply specifying values of the function arguments ξ_1 and ξ_2 as input to a computer program. Thus, the effort required to obtain the parameter estimates would be further reduced and, through iteration, the accuracy of the estimates could be specified for any given situation.

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TABLE I. ESTIMATING FUNCTIONS FOR DOUBLY TRUNCATED NORMAL SAMPLES (Continued)

ξ_2	$Z_1(\xi_1, \xi_2)$	$Z_2(\xi_1, \xi_2)$	$H_1(\xi_1, \xi_2)$	$H_2(\xi_1, \xi_2)$	$\mu_{11}(\xi_1, \xi_2)$	$\mu_{12}(\xi_1, \xi_2)$	$\mu_{22}(\xi_1, \xi_2)$	$\rho(\hat{\mu}, \hat{\theta})$				
$\xi_1=3.7$												
$\xi_1=3.6$												
-3.2 .7332163964 4.1151922441 .6360479047 .0718126693 15350.9+1611212096 22442.6966399399 3280.1255690517 .9998186777	-3.1 .4940434420 3.79984950395 .6575806708 .0679545101 751.2+1242353640 11173.984846144 1663.1414102697 .9997278201	-3.0 .3419860338 3.5680335594 .6707049633 .064198276 41244.5311487151 6254.7046490525 949.25531170746 .9996134755	-2.9 .24658199267969 3.48594222543 .6946036045 24658.199267969 3814.856834834 590.8173840842 .9994730348	-2.8 .1735688146 3.2343205257 .7102758765 .0564096552 15722.7457476548 2483.8191008281 392.928973672 .9993037674	-2.7 .1264537853 3.1022312624 .7242225211 .0528938056 1054.0+90181325798 1701.3146915109 275.2870323231 .9991028150	-2.6 .09328868310 2.9830465508 .7365883911 .0496301235 7353.8292868063 1214.5569349549 201.0852391110 .9988671559	-2.5 .069816193 .2876133391 .7475012335 .0466373712 5297.8265324765 895.9342397003 151.9571025745 .9985935414	-2.4 .05290585442 2.7682645967 .7571107286 .0439186203 3916.3098589393 678.8730768818 118.0855916839 .9982784025	-2.3 .0400120436 2.6682563878 .6955397542 .0414677540 2957.4616959147 525.927251320 9.91867095051 .9979177310	-2.2 .0207908898 2.5714233644 .7179048203 .0392705203 2372.5427333923 414.9733427019 78.1568244337 .9975695355	-2.1 .023923597 2.4770251279 .7931076769 .073151744 1771.3115862127 337.4375476143 62.76723829486 .9970405748	-2.0 .0187604469 2.3849114639 .7848505341 .0355790535 1396.826595783 269.5928670556 52.4361474755 .9965126688
-1.9 .0148479091 2.2935567492 .786061999 .0360452413 111.0281634640 221.0844839266 44.3155121680 .9959154836	-1.8 .0118579117 2.2039253427 .7936487205 .0262957677 892.1247996699 182.8041132789 37.81729101675 .9952420906	-1.7 .0095567413 2.1154750605 .7970398269 .0315136369 720.1198150433 152.2242576049 32.5367253181 .9944765884	-1.6 .0077668437 2.0281186468 .7998324747 .0304832437 584.02994840766 127.4922517860 28.1869297961 .9936118840	-1.5 .0063685769 1.9418104613 .8020718707 .0295902210 475.5428367892 17.75707301986 24.5674969320 .9926313177	-1.4 .0052670338 1.8565354372 .8037963463 .0288217175 388.2417727833 90.5988510767 21.5051559245 .9915172212	-1.3 .0049390962 1.77329105170 .8050381580 .0281662034 317.59553934200 76.1350028500 18.9077639723 .9907485901	-1.2 .0036949666 1.6893144422 .8058242907 .0276133965 260.1407660292 65.1344007146 16.6799629682 .9988004520	-1.1 .0031356254 1.6070741472 .8061766982 .0271541393 214.2314425915 55.5749672031 14.7575683837 .9971431061	-1.0 .0026791996 1.5281722476 .8061136859 .0276802664 174.8205972453 47.1281199864 13.0882509105 .9982412079			
-0.9 .00239198761 1.44644002741 .02656496117 .0264844697 143.02035763100 40.1646916165 11.63123428846 .9830526664	-0.8 .0020660689 1.3680848074 .0264795763 117.6386185683 74.1877364316 10.395412107650 10.2006272008 .99074802330 .9776051513	-0.7 .0017563365 1.29107465395 .0236103671 96.1+192455737 29.1201004749 9.2309082230 8.3399433113 .9742154019	-0.6 .0015494712 1.2155035340 .0219503232 2600.026509 78.6150304642 24.7933023126 8.3635052652 .9702723499	-0.5 .0013772351 1.1414765901 .0259585924 .0263927888 1.0291333419 21.1099489771 7.3635052652 .9702723499	-0.4 .0012331391 1.06990906319 .0259645557 .0264844697 1.02913310320 40.162008812 6.5869400466 .9656474246	-0.3 .0011120462 9.9884676637 .0261056195 .0261056195 42.7279942093 15.24465053241 5.8974045592 .9630175054	-0.2 .0010098608 .9295640389 .0261072909 34.794665818 12.9377571505 6.251344007146 16.6799629682 .9540100604	-0.1 .0009233062 8.8628195953 .0268362149 .0262346075 28.310902239 10.9673197694 6.7413800343 .9466301821	0.0 .0008497437 .7980566214 .0263927888 23.0259293018 9.2854896147 4.2571884860 .9379630208			
0.1 .0007870382 1.7354786183 .7803463210 .0265767576 18.7153062065 7.8511296186 3.8263122673 .9277767304	0.2 .0007345252 1.6751988327 .7757781073 .0257815664 15.2109876364 6.52193040118 3.44282952053 .9150061767	0.3 .0006875652 1.6173285511 .7708937634 .0260070868 12.3870124664 5.5849631049 3.101519018 9.917450512	0.4 .0006482088 1.59379751340 .0272224927 10.1001411811 4.709216262515 2.7977785547 1.7152545666 .9846746754	0.5 .0006144173 .5052938249 .0274607108 8.2564846730 3.9561476789 2.5275912469 .8660726887 .96607026887	0.6 .0005853878 1.4592139367 .7538069863 .0276868651 6.7765369773 3.320781841984 2.3927815928 .8436865450	0.7 .0006604940 4.1919833382 .7674038888 .0279064674 5.5863608202 2.7831601929 2.0736507162 .8177224291	0.8 .00053930361 3.3676117056 .7466501579 .02810715310 4.6360787324 2.3299093726 1.6838288399 .7808546871	0.9 .0005026712 3.2616159841 .7355841020 .0298454543 3.8778772795 1.5466731345 1.7152545666 .7544101300	1.0 .0005049472 2.876368252 7.261621536 .0284312439 3.27462894550 1.6280645655 .1.5656402881 .7167409166			
1.1 .0004915152 .2520777458 .7184195353 .0285416772 2.7948130831 1.3512062627 1.4329423687 .6751976035	1.2 .0004807043 2.1946328084 .7190415905 .0285965021 2.416654891 1.1203529487 1.3153531888 .6301630976	1.3 .0004703623 1.897576836 .7021425357 .0286297992 2.1137176160 0.931651373 1.2111883271 .5822672326	1.4 .0004721519 .1629003112 .6953957666 .0285983640 1.8766407126 0.7713407303 1.1193463225 .53235645680	1.5 .0004552428 1.3880057849 .6849325881 .0285120581 1.6884092579 0.6372123377 1.0376105840 .6814326722	1.6 .00044945058 1.1736500558 .6760536704 .0283684980 1.540864991 1.0523764215 1.9657231094 .4205815002	1.7 .0004446640 .0984467023 .6703650957 .0281841952 1.424047515 1.417256516 1.0235156982 .3085890267	1.8 .0004406609 .0894174631 .6719161669 .0281791092 1.3323243486 0.3538796919 1.065776246 1.4585654593	1.9 .0004373877 .0765632831 .6847725187 .0275990288 1.2620521523 0.2911420097 0.7975788546 .2884644919	2.0 .0004347170 .0595235977 .6395053964 .02723594104 1.2035293818 0.2353993029 0.7546275072 .2470077092			
2.1 .0004425254 .02902883456 .5852029776 .0242570177 1.0450809094 0.615523400 .5918382924 .0782650884	2.2 .00044038165 .0359786993 .5852029776 .0242570177 1.0450809094 0.615523400 1.0450809094 .0782650884	2.3 .0004429318 .0286372337 .5680752813 .0231042029 1.0268467662 0.6369713598 0.5611801566 .0486166608	2.4 .00044283378 .0225820821 .6029526157 .0236126197 1.0750689936 0.5987257050 0.5521187672 .0374644000	2.5 .00044274809 .0176397389 .5939980229 .0248184889 1.0582337062 0.478631189 0.6100329579 .0974068966	2.6 .0004268157 .0136480566 .5852029776 .0242570177 1.0450809094 0.615523400 0.5918382924 .0782650884	2.7 .0004263042 .0104583029 .5765574974 .0236837052 1.0348299097 0.6079451768 0.5763249539 .0621094662	2.8 .0004259145 .0104583029 .5765574974 .0236837052 1.0348299097 0.6079451768 0.5763249539 .0621094662	2.9 .00042456203 .0059643036 .5957666662 .0225529210 1.0268697836 0.5821484364 0.5221187672 .0374644000	3.0 .0004245004 .0044383181 .5916398630 .02194464616 1.01946464616 0.5104839363 .0284804578 .0284804578			
3.1 .00042452376 .0032703360 .5436995202 .0213728479 1.0123031281 0.5154666330 .5522753139 .02095790909	3.2 .0004251182 .0238598494 .5359477005 .0208105034 1.0095260508 0.5093024829 0.5289433289 .0150436723	3.3 .0004250316 .0017235880 .5283859205 .0202595963 1.0074233393 0.5075240401 0.5238310896 .0103573963	3.4 .0004249693 .0012327674 .5210129862 .0197292862 1.0058428101 0.4983282292 0.5197131482 .0066842869	3.5 .0004249249 .0008072979 .5138266590 .0192008516 1.0046641600 0.2076317679 0.5164303103 .0038369423	3.6 .0004248937 .0006120653 .5068236751 .0186943971 1.0037926328 0.0011880935 0.5138412863 .0016542995	3.7 .0004248719 .0004248719 .5000000000 .0182040896 1.0031539681 0.0000000000 0.5118220679 .00000000001						

TABLE I. ESTIMATING FUNCTIONS FOR DOUBLY TRUNCATED NORMAL SAMPLES (Continued)

ϵ_2	$Z_1(\epsilon_1, \epsilon_2)$	$Z_2(\epsilon_1, \epsilon_2)$	$H_1(\epsilon_1, \epsilon_2)$	$H_2(\epsilon_1, \epsilon_2)$	$\mu_{11}(\epsilon_1, \epsilon_2)$	$\mu_{12}(\epsilon_1, \epsilon_2)$	$\mu_{22}(\epsilon_1, \epsilon_2)$	$\rho(\hat{\mu}, \hat{\sigma})$
$\epsilon_1 = -3.5$ (Continued)								
$\epsilon_1 = -3.4$								
1.6 1.7 1.8 1.9 2.0	.00109235049 .009136107 .009054256 .008969933 .008932112	.1173804096 .0986598927 .0819123504 .0875719685 .0552610173	.6634398930 .0543180227 .0303219404 .0300426500 .6264785807	.03059404184 .0303219404 .0300426500 .0296104008 .0292031692	1.5604270082 1.44238267569 1.3321751554 1.25601749221 1.2026464631	.5264278802 .47319603462 .4533481546 .2880301152 .3338491907	.9785754836 .91386767410 .8569811101 .8070355551 .7632820607	.4287666788 .3786792105 .3307017842 .2856181010 .2439747905
2.1 2.2 2.3 2.4 2.5	.0008887666 .008851959 .008823504 .008801021 .008783412	.0447942396 .0359823549 .0286409480 .0225849249 .0176419552	.1159712917 .5078773535 .5986623280 .5895415555 .5805393977	.0287274784 .0270888603 .0276536055 .0270562196 .0264594416	.15159712917 .11424592153 .13571912803 .13759321083 .13951302580	.1889908906 .1519303121 .1213667629 .07952850032 .0757223027	.7259463287 .6172725847 .6627854106 .6377515430 .6161325980	.2061451094 .1722674201 .1423450521 .1162413485 .0937334085
2.6 2.7 2.8 2.9 3.0	.0008769744 .0087952933 .0087512123 .0087451783 .0087404599	.0136497687 .0104596313 .0078330166 .0075340547 .0079130466	.5716765911 .5622703697 .5943435036 .5466796044 .5791304666	.0258948631 .0251977476 .0245566650 .0239163728 .0232987442	1.0466940628 1.035431744 .0453027478 .0254019171 .0183093494	.0589405466 .0453027478 .0347933679 .0254019171 .0245934199	.5976937030 .5819329998 .5675910280 .5377515430 .5479654199	.0745400107 .0583472098 .0448214845 .0336577584 .0245232200
3.1 3.2 3.3 3.4 3.5	.0008767314 .0008734861 .0008730380 .0008731800 .0008730089	.0032727447 .0023862830 .0017230803 .0012329214 .0008730889	.5299938465 .5221622691 .5164806809 .5071942404 .5000000000	.0226557893 .0220384140 .0214371572 .0208517498 .0202934363	1.0136772573 .0117628171 .0047653698 .0027700124 .0000000000	.0126768171 .0121137972 .00647633698 .0027700124 .0000000000	.5401945132 .533137972 .5246232279 .5244422697 .5211791414	.0171311016 .0112119132 .0065226831 .0028460178 .0000000001
$\epsilon_1 = -3.3$								
-2.9 -2.8 -2.7 -2.6 -2.5	.8659598639 .5555382316 .3936746438 .2849550033 .989202875	.3+8933838569 .3+5684103293 .3+3293247716 .3+1411092614 .2+9846906645	.6251520140 .6451585272 .6633569604 .6798071773 .6945884700	.0734509406 .0700842149 .0666151463 .0631561284 .0597926847	125494+539513746 6070+1128912061 32945+971329223 19450+641983149 12238+8115969537	20079+5906234526 9933+9727141120 9446+8877515052 3309+2290857033 2129+3501499898	3214+2356625844 1617+0112520075 914+5107642517 563+7555776356 371+5976556817	.09078400734 .0987454658 .0905345500 .0923638128 .0991561193
-2.4 -2.3 -2.2 -2.1 -2.0	.1567587874 .1186288366 .0982879596 .0703020842 .0597738803	.8+889569911 .2+7271151246 .2+6148634519 .2+5094062370 .2+4088911194	.7078017964 .7195579200 .7299704161 .7391506517 .7472042407	.0565857337 .0535752939 .0507842650 .0482230433 .0458923774	8.92+5478720114 5165+8534759568 3951+282899464 2878+2627345351 2141+2921676756	1441+175703130 1015+575031307 739+3965476313 552+9205160666 422+6973957069	.257+2149403227 .185+815764831 .138+8424642524 .106+6720161118 .83+8771971998	.998+375197 .998+132760 .998+621523 .9978661600 .5974006650
-1.9 -1.8 -1.7 -1.6 -1.5	.0346191046 .0346193261 .0278602764 .0226251509 .0185378987	.2+3120741188 .2+2181129316 .2+1264345159 .2+0366511793 .1+980538720	.7542290855 .7631614965 .7625544626 .7699855398 .7698513066	.0437865904 .0418957979 .0402673627 .0387084971 .0373844383	1520+8409323496 1604+442225296 9864+77602678 758+2736113607 599+4707127597	379+1128017504 260+1806289904 208+1847858989 166+5216585075 137+1092430436	.67+247662174 .54+8057920955 .49+289529767 .37+8611283049 .21+9833468778	.9968839174 .9962469456 .9959324499 .9947284424 .9937999031
-1.4 -1.3 -1.2 -1.1 -1.0	.0153223490 .0127739346 .0107390987 .0091053504 .0077831746	.1+8618250235 .1+7765112584 .1+6925207765 .1+6097951293 .1+5283810325	.7767486628 .7791720311 .7810146107 .7823087918 .7830842259	.0362217299 .0352072361 .0342861619 .0309744201 .0329340961	476+8271754276 381+232898073 .306+3121923469 .246+3823462456 .198+2803671722	113+1489674752 93+2328656724 .77+816397867 .54+9496883841 .54+3790747938	.27+2440764285 .23+3762430700 .20+1841125437 .17+5205666414 .15+2782259370	.9927362652 .9915168891 .990117218 .9885079132 .9865945076
-0.9 -0.8 -0.7 -0.6 -0.5	.0067069330 .0052852586 .0050998680 .0044985263 .0039891075	.1+4482942577 .1+3695604139 .1+2922983882 .1+2165202967 .1+1423252127	.7833650701 .7831712203 .7825188360 .7814207963 .7798720510	.0329795992 .0319571165 .0316033746 .0313291348 .0311272812	160+6969924486 130+0204382151 .105+2635879713 .85+2407846540 .69+0248924190	45+644394271 .38+8303636451 .32+3215318184 .27+243292630 .22+9737421957	13+3753520498 .11+7422599467 .10+3512959731 .9+1431492361 .8+294774570	.9845129584 .9820376701 .9791656451 .9758651373 .9719350283
-0.4 -0.3 -0.2 -0.1 D.0	.0035795213 .0032278031 .0029310402 .0026796792 .0024661001	.2+0098022264 .2+990463978 .2+0301607218 .2+0362495263 .1+7984225847	.7779257650 .7755242144 .7727407245 .7708998169 .7709872051	.0309910735 .0309140272 .0308998169 .0309121718 .0309747867	55+8818215891 .45+2271457249 .36+5921304295 .29+5987145688 .23+9408003434	19.7720235596 16+344017709 .13+8174372057 .11+6453084685 .9+7819787639	.7+1304871015 .5+3815659118 .5+6812806915 .5+0660762818 .4+5248260599	.9673893084 .9820376714 .9582859616 .9478846341 .9398653223
0.1 0.2 0.3 0.4 0.5	.0022840705 .0021284699 .0019952597 .0018810071 .0017829166	.2+022840705 .2+654662680 .2+509086808 .2+621716965 .2+594068537	.7618551583 .7617203222 .7625507205 .7621716965 .7416342626	.0301701247 .03052072361 .0342861619 .0313906968 .0316600834	19.3694975926 15.6280201117 .12.7127683109 .10.3265405870 .8.4127865157	8.2318015274 .9.328257899 .5.8126706531 .3.2543466233 .4.8863397741	.13+3753520498 .11+7422599467 .10+3512959731 .9+1431492361 .8+294774570	.9845129584 .9820376701 .9791656451 .9758651373 .9719350283
0.6 0.7 0.8 0.9 1.0	.0016986521 .0014261845 .0013929780 .0013647915 .0013406926	.1+4493603955 .1+41709227 .1+38685395 .1+35108087 .1+3264415108	.7358456462 .7291508149 .7255072025 .7247219238 .7157897179	.0323423306 .0321109975 .0313906968 .031201718 .0322224235	6.8812158565 .56581444942 .4685526327 .390052393112 .3.924381568	3.4+193829893 .2+857722358 .2+3849226795 .2+2482084590 .1+9816167048	.2+3778339994 .2+1507822201 .1+9491582168 .1+2428084590 .1+7711525170	.845+129584 .8292857899 .80830365491 .753254466233 .753610170
1.1 1.2 1.3 1.4 1.5	.0014261845 .0013929780 .0013647915 .0013406926 .0013209106	.2+52145961 .2+195021263 .2+081921254 .2+0619092721 .2+1088398766	.761151909 .757161677 .753379567 .7509280975 .7416342655	.0322432306 .0323374419 .03211270791 .0321705241 .0320007338	6.8812158565 .56581444942 .4685526327 .390052393112 .3.924381568	1.37373873886 .1+13887767699 .1+9492304303 .1+2428084590 .1+773885852	.4+4749148595 .4+319194702 .3+9216031282 .2+8705694922 .1+0622481505	.845+129584 .8292857899 .80830365491 .753254466233 .753610170
1.6 1.7 1.8 1.9 2.0	.0013041236 .0012901500 .0012785901 .0012690967 .0012613399	.2+1173934668 .2+0947064499 .2+081921254 .2+0619092721 .2+0552669179	.6567821314 .6476116677 .6383301451 .6298990875 .6196285965	.0317703879 .0314790213 .0311276395 .0307186731 .0302005864	1.5403128543 .142367107091 .133206856526 .1265178438 .120359656077	.527+20798190 .4319194702 .35287814868 .3+41394878 .2+322666759	.2+776788695 .2+1507822201 .1+9491582168 .1+843615718 .1+67722481505	.9273364373 .9177536181 .89262860659 .8871757206 .8678468238
2.1 2.2 2.3 2.4 2.5	.0012550630 .0012450509 .00124490012 .0012428286 .0012403932	.2+0447989982 .2+039938139 .2+0266438687 .2+025873008 .2+0176438073	.6102647931 .59169445905 .59169445907 .5825268147 .573409376	.0297440921 .0285974778 .0279750002 .0273313489 .0273313489	1.15952230584 .1+076399584 .1+0764598140 .1+0764598140 .1+0764598140	.187503561612 .1+0976399844 .1+0961031282 .1+0764598140 .1+0764598140	.7+3071338336 .6+695878381 .6+6772248036 .5+1397206655 .5+0908860000	.2037025786 .1697191042 .13971706655 .1139442423 .0908860000
2.6 2.7 2.8 2.9 3.0	.0012384089 .0012369243 .0012357932 .0012349394 .0012343012	.2+0136511995 .2+0094761899 .2+0093840999 .2+0059656733 .2+0044393368	.5645978682 .5595265505 .5595265507 .5382568147 .5307492132	.0266706170 .0253258541 .0246530202 .0239862083 .0239862083	.1+04693035484 .1+029039583 .1+0265174864 .1+0183163232 .1+0183163232	.2+569626067 .2+5294463434 .2+5043291899 .2+4863615718 .2+4749148595	.6+6186160506 .5+5892560012 .5+5545437356 .5+5272467361 .5+515969860	.0717605934 .0589256012 .05545437356 .0470077242 .0216841729
3.1 3.2 3.3 3.4	.0012338287 .0012334823 .0012332309 .0012330501	.2+00123710863 .2+0023865322 .2+0017239832 .2+0012330501	.5227634988 .5149768106 .5073894399 .5000000000	.0233292418 .0226852141 .0220565300 .0214494667	1.0+147552945 .1+0120445748 .1+0099464482 .1+0084556388	.+5437323172 .0061688560 .0026940598 .0000000000	.0142875157 .0083657185 .0036751651 .0000000001	

TABLE I. ESTIMATING FUNCTIONS FOR DOUBLY TRUNCATED NORMAL SAMPLES (Continued)

ξ_2	$Z_1(\xi_1, \xi_2)$	$Z_2(\xi_1, \xi_2)$	$H_1(\xi_1, \xi_2)$	$H_2(\xi_1, \xi_2)$	$\mu_{11}(\xi_1, \xi_2)$	$\mu_{12}(\xi_1, \xi_2)$	$\mu_{22}(\xi_1, \xi_2)$	$\rho(\hat{\mu}, \hat{\sigma})$
$\xi_1 = -1.9$ (Continued)								
-0.4	+20773380917	1.16592211171	.52778798564	.0682443601	1.974735745488	.904022670421	42.1747886286	.9833890609
-0.3	+18556468063	1.0792381327	.6290211838	.0673705256	1.974729574360	.661545274703	33.1379654613	.9789867979
-0.2	+16737175003	0.9974975067	.6293409566	.06656585501	0.94551837378	49.7937461020	26.3047975774	.9735171494
-0.1	+15298615410	0.9200151040	.6289163485	.0658340519	1.162070695824	.376317112153	21.1577528026	.9667119701
0.0	+13992279235	0.8465017999	.6277509914	.0651608335	52.005245223	28.6738577772	17.2171654126	.9582566390
0.1	+12837872623	.7766460558	.6258636352	.0655613280	38.0429514129	21.995189375	14.1584997575	.9477242296
0.2	+11918378211	.7102852778	.6238250020	.0639666168	28.0230240702	16.9621276793	11.7520271329	.9344118303
0.3	+11136521338	.6473033623	.5200281166	.6342673718	20.7845315319	13.1347350578	9.8431447561	.9182993161
0.4	+1046996525	.5876290016	.6161176743	.0629108324	15.5278734367	10.2006679203	8.308986020	.8980470144
0.5	+0990059910	.5312221863	.5115765893	.0624074105	11.6950455794	7.9362144306	7.0663596448	.8730006471
0.6	+0941362415	.47806429184	.50619045121	.0619045121	4.8921661373	6.1783786137	6.0518616197	.8422743288
0.7	+08996850120	.42814409777	.6007017125	.0613899836	4.8380462275	4.8070674028	5.2176592993	.8476459108
0.8	+0864016222	.3814601738	.5544227587	.0608517462	5.3320198612	3.7323151351	4.5272745167	.7597542938
0.9	+0833095946	.3380047883	.5176236398	.0602780997	4.2297255709	2.8841939871	3.0576365186	.7066483351
1.0	+0807451869	.2977165151	.5030386351	.0556850826	5.4161920748	2.2277077748	3.718442069	.6454073133
1.1	+0782375795	.2607080495	.5726925253	.0589816548	2.9232627567	1.5967358577	3.0677580974	.5766463216
1.2	+0766348511	.2267962275	.5644240714	.0582036308	2.3675975410	1.4042959351	2.7266448870	.5017919861
1.3	+0750338473	.1959658171	.5595981199	.0574273346	2.9711270157	.9504294462	2.0477108647	.4240115567
1.4	+0736620231	.1681335511	.447135865	.0565377738	1.86161310467	.6867581646	2.9212133617	.3429034334
1.5	+0724546739	.1431160299	.5380439348	.0556597976	1.6734549576	4.611180564	1.9828556942	.2641188669
1.6	+0715951442	.12102865650	.5287332855	.0565207937	1.55277395860	.3163257656	1.8060799993	.188980491
1.7	+0708045127	.1014851407	.5192550894	.0533953346	1.45959451430	.1856907755	1.6510725581	.1193020714
1.8	+0701508492	.2844667941	.5096005552	.0521970728	1.4538523166	.0820285736	1.5192277749	.0561418555
1.9	+0696139618	.0696139618	.500000000	.0509326139	1.3596804131	.000000000	1.4074102501	.0000000000
$\xi_1 = -1.8$								
-1.3	-1.29790255249	2.8153133923	.5634242644	.0802536656	28135.704535187	9136.369616289	2972.0736161822	.9941138652
-1.2	-1.976093713	2.4537231132	.5731437633	.0791767723	12/9.8734642658	4296.736879095	1449.7299969822	.9984251786
-1.1	-1.7915934333	2.1842934655	.5818561117	.0780417214	6471.2011161168	2259.579167288	792.1824745084	.9979774391
-1.0	-1.6433098410	1.9716508701	.5895/37136	.0768801042	3351.093725851	1288.706726211	470.360941558	.9971342608
-0.9	-1.529795516	1.796278327	.5763130254	.0757182670	20646767535220	.7814453260862	297.5370861627	.9960233368
-0.8	-1.4487714803	1.6446756765	.6020795836	.0747577384	12/1.9787062963	496.9748731716	197.4152692732	.9946731549
-0.7	-1.383191913	1.5155505000	.6694602624	.0734739733	7/8.3365247666	326.2511117114	136.4947711052	.9929324949
-0.6	-1.3312740643	1.3902689498	.6108929154	.0724012425	519.6216511794	223.55031340405	9.70813651883	.9979735399
-0.5	-1.2961139159	1.2914764655	.6195113082	.0714241184	346.6611991041	156.0610153001	71.0752108092	.9879738727
-0.4	-1.255705701	1.1911721096	.6161178125	.0704962639	355.6267615000	11.212316061090	54.1552811577	.9845211079
-0.3	-1.207632727	1.1017791778	.6175164740	.0692761652	162.7155232916	80.01161313514	4.1666115058	.9801197659
-0.2	-1.166616612	1.0166608642	.6101660862	.0688161612	113.8588975387	59.25709624282	37.7316671112	.9747541764
-0.1	-1.1869705656	.9356771625	.5718952635	.0670711092	80.5648850130	44.67070116296	24.7316470449	.9670702732
0.0	-1.172565749	.8596632955	.6194251979	.0673184187	95.55332975430	33.0913305791	20.6716774612	.9509187333
0.1	-1.1566799949	.75776446425	.6152181872	.0676403105	41.4664702271	25.0410778268	16.8047607576	.9484315707
0.2	-1.1453809494	.7197170707	.6127961598	.0661395673	30.1082051593	19.0665513827	13.8076929183	.9525139437
0.3	-1.1356575979	.5593260826	.6096133310	.0655696621	22.0929722998	14.4559848324	9.1826116160	.91826116160
0.4	-1.1274435352	.5944717343	.6058625959	.0650207115	16.2484811412	11.1953630798	9.5889499537	.8969023460
0.5	-1.1204367463	.5370767951	.6016464608	.0644798251	12.0913292502	8.60976496266	8.02892532	.8703636308
0.6	-1.1144659461	.4830626295	.5964117966	.0639368309	9.0191335237	6.6249579530	6.8830108253	.8376432087
0.7	-1.1093319337	.4324211694	.5907648097	.0633791348	6.9229970398	5.0998263055	5.89764633967	.7970867071
0.8	-1.1049570501	.3851163925	.5845533138	.0626747415	5.3527070707	3.9045390194	5.0867732353	.74862610411
0.9	-1.1012161917	.3411307677	.5778968670	.0621719101	4.2156548261	2.97913618686	4.4717813237	.6907308558
1.0	-1.0980242661	.3354034047	.5705693556	.0614997603	3.392719521	2.26266252926	3.8616296434	.6734482355
1.1	-1.0933039755	.26297391616	.5628709220	.0607683749	2.7986186602	1.6969062998	3.3969139921	.5849851549
1.2	-1.0929194041	.2273232525	.5545762244	.0635692464	2.36967327079	1.2666268215	3.0063096204	.6670231193
1.3	-1.0912350473	.1975956171	.5462103190	.0609357992	2.0018194758	.9780900758	2.6780808131	.8420869849
1.4	-1.0893795923	.1695066949	.5374646265	.0581438263	1.8415548469	.62348161329	2.3992130355	.2965731094
1.5	-1.0879940369	.1443474499	.5283765262	.0571093224	1.6847471775	.4077094469	2.1638709321	.2135346835
1.6	-1.086820318	.1219888341	.5190703523	.0559946402	1.5737615251	.230307410435	1.9627774764	.1356241876
1.7	-1.085616570	.10226623773	.5095940807	.0547995494	1.495763312	.1046688373	1.7915255457	.0639384866
1.8	-1.0850628274	.0850628274	.500000000	.0535319387	1.4413919480	.000000000	1.6452062381	.0000000000
$\xi_1 = -1.7$								
-1.2	-1.1339498558	2.7542477530	.5594042055	.0805457276	74550.7774671025	8510.9075010618	2062.67415464070	.9984886087
-1.1	-1.1323650516	2.3913634659	.5683811428	.079532159	11/79.4811763272	3929.7511646302	1643.376440890	.9984343636
-1.0	-1.1262362206	2.1028711214	.5673756949	.0785485622	5580.861780635	2091.7404086167	787.6662115756	.99767695466
-0.9	-1.1274217307	1.9076441284	.5633398252	.0775101042	3.942.8611352367	1156.1212364665	467.01181556230	.996681611490
-0.8	-1.1261392802	1.7137613524	.5694605644	.0764541559	1.761.576155474	71.73235074252	.99547863809	.99547863809
-0.7	-1.1246436737	1.5184810319	.5645565454	.0754205070	1.66.3473588561	454.65586585717	.9947354625	.9947354625
-0.6	-1.1040546706	1.4547027823	.5609811748	.0744225441	6.694.472112120	.798.636364990	138.2949517260	.9916712720
-0.5	-1.1062819040	1.3372184194	.5602195951	.0734676530	4.12.6277127448	.202.0543434464	.98.6141040404	.9808187305
-0.4	-1.1089485539	1.227716773	.5604914658	.0729654429	2.96.1394768724	.70.01171188771	.19.9452116877	.9846611658
-0.3	-1.1086448848	1.1314688646	.5603371904	.0719177543	21.3400370124	.16.1233310321	.12.0961361434	.9845511507
-0.2	-1.1050442768	1.0295672462	.5566510163	.0706045411	1.31.1505455109	.14.0030476819	.9.1765614317	.9847354625
-0.1	-1.1043034002	.9646505649	.5520611111	.0693843664	1.4.1893463521	.1.4054363521	.4.9817310316	.6.694663126
0.0	-1.1045849339	.9462103085	.5501366149	.0686554515	1.2.43661087050	.9.3781505000	.9.3723612171	.5.954465437
0.1	-1.0984216706	.8461802716	.5488610146	.0684784411	9.277839575447	.7.12.05430447	.1.0.10337785	.8.3311133727
0.2	-1.0911498734	.7411654662	.5403901798	.06761204467	6.086156662	.6.17.73102372	.1.1976028252	.6.7870625272
0.3	-1.0953336565	.6476626062	.5366511870	.0676045851	7.1.3576125114	.5.18.92484657	.2.9.36639664	.5.316339664
0.4	-1.0752426306	.56034956417	.5306711111	.0663843664	6.4.1893463521	.5.1054363521	.4.9817310316	.6.694663126
0.5	-1.0663635641	.45675161617	.526987626	.0658580556	1.37.7142120167	.6.23.311525117	.5.954465437	.5.954465437
1.1	-1.1147263193	.20574846196	.5591206170	.0647608550	2.7694359777	1.6.611747216	.3.7.19186769	.5.1268290466
1.2	-1.1119145756	.2710735046	.5577116151	.0646240467	2.4.15169976	.1.1875021521	.1.1.3614187467	.4.236356722

TABLE I. ESTIMATING FUNCTIONS FOR DOUBLY TRUNCATED NORMAL SAMPLES (Continued)

ξ_2	$Z_1(\xi_1, \xi_2)$	$Z_2(\xi_1, \xi_2)$	$H_1(\xi_1, \xi_2)$	$H_2(\xi_1, \xi_2)$	$\mu_{11}(\xi_1, \xi_2)$	$\mu_{12}(\xi_1, \xi_2)$	$\mu_{22}(\xi_1, \xi_2)$	$\rho(\hat{\mu}, \hat{\sigma})$
$\xi_1 = -1.6$								
-1.1	1.3716491423	2.6939641470	.5553699907	*080819B497	21724.9705380767	7905.8968527329	2953.9191535509	.9988344100
-1.0	1.0680256799	2.3298683796	.5635954672	*0799451735	9519.3532577697	3692.4400006143	1437.4980731438	.9981733108
-0.9	.8581163493	2.0585141097	.5708603423	*0790239931	4762.9786654295	1926.5181201904	783.4815048720	.9972852727
-0.8	.7062497402	1.84465099556	.5711747309	*0780813468	2578.5340036992	1089.4694097322	463.9175176649	.996121014
-0.7	.5926386539	1.6683407701	.5829532043	*0771378396	1480.710132069	654.6705970412	292.6125933319	.9745809463
-0.6	.5054404231	1.5164269641	.5970134770	*0762097950	689.7311956578	412.3534463397	193.0693355259	.9952865665
-0.5	.3371466895	1.3875136293	.5905735275	*07509305133	553.9729879490	269.5080387307	133.8379420410	.9700446243
-0.4	.3827773826	1.2782856688	.532597623	*0744455913	254.8282322347	181.5124880105	95.4425291991	.9667677536
-0.3	.3389076251	1.1652342687	.5408444149	*0732633547	232.5225791515	125.3924246848	67.5643525457	.9526566359
-0.2	.311112530	1.0685494243	.564083919	*0726477557	159.2215676915	88.2178640279	52.5010494374	.9771304186
-0.1	.2736266645	.9792287016	.582656542	*0712596107	105.2441267795	63.1045455981	40.1561464229	.9707266857
0.0	.2491479295	.8960553407	.5996576860	*0714149843	72.2762315550	45.5472103128	31.3172652261	.961472452
0.1	.2288489290	.8184106917	.5942815296	*0707559635	50.1936182870	33.4502640532	24.7757748400	.5003709061
0.2	.2119491499	.7456695460	.5981586688	*0702152819	35.2074936604	24.7416904817	19.8710187950	.9350494333
0.3	.1969782927	.6772857390	.5893118410	*0695162888	24.932750715	18.4381539397	16.1323721659	.916623778
0.4	.184676474	.6131647477	.5857644968	*0689178182	17.8379440821	13.7147979786	13.262912015	.892126684
0.5	.1742221633	.5529852261	.5815413987	*0683198340	12.8952982200	10.2539310355	10.3813642320	.8017140668
0.6	.1653196706	.4966477376	.576690605	*0677110737	9.4425861110	7.6465861261	9.193651012	.822090311
0.7	.1577289389	.4440237156	.5711716182	*0670798474	7.0268928962	5.7513527534	7.7610703938	.773924323
0.8	.1512523135	.3959274842	.5659045053	*0664414056	5.3333641617	4.2387664222	6.677520021	.7140468630
0.9	.1457227693	.3459587515	.5984686955	*0657333004	4.1466568469	3.1443930288	.6.691368339	.6244239380
1.0	.1410227903	.3076373059	.5130210749	*0649357741	3.3167507643	2.2461847657	4.9008577655	.5566057833
1.1	.1370180362	.2691179100	.5436704112	*0641021574	2.7308441405	1.9893078921	4.2675565790	.4675025774
1.2	.1336184665	.2339212174	.5361508190	*0631942624	2.3375152682	1.0429300697	3.3242556779	.3695173345
1.3	.1307411662	.2019402272	.5271554807	*0622057485	2.0616793987	.7045334558	3.3047488182	.2697102362
1.4	.1283146469	.1732066590	.51536933U0	*0611324385	1.8736032554	.405808792	.2.9286660605	.1729701917
1.5	.1262769292	.1474482617	.50529957U2	*0599725594	1.7469084645	.1762447840	.2.6300773732	.0622236649
1.6	.1245739647	.1245739647	.5000000000	*0587268860	1.6628882718	.0000000000	2.3609929306	.0000000000
$\xi_1 = -1.5$								
-1.0	1.4101289239	2.6344676616	.5913225247	*0810757247	18154.0686673074	7302.9452505102	2945.0044156680	.9980407455
-0.9	1.0946001341	2.2693272001	.5878682235	*0802020576	885.6187291154	3395.5198773363	1432.0609336089	.9978583376
-0.8	.8929279527	1.9972089149	.5653129683	*0794669138	415.5377795817	1763.694768748	775.6313554248	.9967971256
-0.7	.7396394837	1.7827144342	.5936063119	*0786222750	2156.6327218584	492.5649583090	61.8738385118	.995918C55
-0.6	.62643439151	1.6063203776	.5755817083	*07776760259	1227.9807819155	533.3570581127	200.4632048110	.9951577679
-0.5	.5359731155	1.45643883208	.5793553948	*0769420979	731.2592353534	371.6459644880	192.3C25212411	.9910735777
-0.4	.4667246256	1.3280458182	.5282445941	*0761030711	450.9969553636	241.5228820042	98.876155114	.9837529776
-0.3	.410800196	1.2096741713	.5744213352	*0753486059	285.7367265377	161.631171313	74.3975336561	.9350595276
-0.2	.3659380801	1.1048492097	.5856542978	*0754994464	155.4684269198	110.7141011119	65.1142611101	.9785192940
-0.1	.32955255473	1.0901202481	.5858109280	*0738842682	122.4880494956	77.3830860588	51.8083386360	.9714072747
0.0	.298937228	.9209347006	.5853661648	*0732066209	82.1247676121	54.8486425787	37.629223874	.9527461339
0.1	.2730896096	.8391865328	.5841961230	*0725456206	55.7734539073	30.4268195853	30.8426263554	.9505023720
0.2	.2517406806	.7630808470	.5821528633	*0719120153	38.3107602119	28.5843720307	.24.3506817266	.9350595080
0.3	.2350147046	.6920429494	.5794287277	*0712932675	26.5995837663	.20.8668805051	19.5518931164	.9150114845
0.4	.2200380500	.6265558666	.5759062269	*0706799904	18.6725050255	15.3079745749	15.8665959253	.880880782
0.5	.2073425199	.5361514041	.5718635579	*0706062075	13.2697371748	11.2411646753	13.0252242944	.8550419463
0.6	.1965545548	.5096981883	.5670744669	*0694284509	9.56699351575	8.25215211181	10.8025610754	.8116136866
0.7	.1873728796	.4517372205	.5616525723	*0687677937	7.0295929923	6.0342C81726	9.0455730205	.756704762
0.8	.1795520316	.4016030404	.5565629839	*0680685107	5.28431976704	4.3772472438	7.6427244458	.6887947516
0.9	.1728900368	.3519190888	.5694012396	*0673193161	4.0863841344	3.1339283792	5.5129265593	.6073204852
1.0	.1672192608	.3124067079	.5161292011	*0670062075	3.2679325993	2.1954674787	5.5946041330	.5135154357
1.1	.1623990639	.2731597158	.5343228262	*0656298513	2.71001306492	1.4840111369	4.84365C1038	.4102556133
1.2	.1581306395	.2373555375	.5267297674	*0646720576	2.3394585987	.9288666365	4.225261371	.3022615101
1.3	.154526851	.204890202	.5171843132	*0636320961	2.0483718890	.52410964550	.7.1310593036	.1548952828
1.4	.1519381801	.1768465484	.5070680799	*0629305056	1.9190882341	.2335241944	.3.285553948	.1529265445
1.5	.1494918614	.1494918614	.5000000000	*0618284906	1.8131563561	.0000000000	2.9207675974	.0000000000
$\xi_1 = -1.4$								
-0.9	1.449394034	2.175780098	.5472627272	*0812100393	18324.177004983	6701.687048447	2908.5279472250	.9983464936
-0.8	1.1420569140	2.0077035343	.5359609328	*0804133154	6776.0046661781	3101.7187340403	.9977414156	.9977414156
-0.7	.9289701403	.1.3639757124	.5597357541	*0798767464	3336.7208605936	1602.5697769805	776.114112023	.9961640620
-0.6	.773795144	1.7221224897	.5645626208	*0791214190	1775.8104680640	897.3088184479	458.9048593201	.9942421405
-0.5	.6573311420	.1.5456316068	.5685550391	*0783635665	1L1.45195153503	.533.2565115759	.298.51P3919432	.9921253653
-0.4	.5657330076	.1.3959059511	.5711627056	*0776143632	.593.0151785030	.131.8800131435	.19.8029242034	.99802735754
-0.3	.496855153	.1.2656734715	.5728391307	*0768821475	.359.9888958767	.214.1762515152	.13.3675824655	.9849556495
-0.2	.44039361010	.1.1501809349	.5751792797	*0761716140	.225.6507491009	.192.2548477725	.9.4563396816	.9795950838
-0.1	.3946266409	.1.0462212109	.5756596846	*0754848167	.144.5652114643	.96.6810442842	.68.3644753994	.9725106265
0.0	.3571373736	.5515769044	.5620403352	*0748210822	.94.2804548839	.66.915344004	.51.202253435	.86315180154
0.1	.3261530516	.56464862763	.5743111836	*0741773627	.02.4007203378	.44.9885670410	.39.15745365	.9507856981
0.2	.3003541615	.5794433943	.5724501607	*0739456139	.41.8514612437	.33.3545456498	.30.4481689893	.9436605311
0.3	.2784717639	.5712014770	.5653840974	*0729275759	.28.4079749158	.23.6622788209	.20.0529743291	.9126770376
0.4	.2605657352	.56408430782	.56650523745	*0723066066	.19.5231316843	.17.1517316217	.19.284165596	.8839453616
0.5	.2451711785	.57646892450	.5624641793	*0716742551	.13.6057984157	.12.3473113123	.15.6510592935	.8461341186
0.6	.2321391244	.51.66351225	.5575752010	*0710215448	.9.645017439	.8.4721719911	.12.8690118126	.7968227465
0.7	.2201701881	.4410413374	.5523956474	*0703367642	.6.9872739175	.6.3341384668	.12.6609412029	.7338982004
0.8	.2116624510	.409528876	.5544270742	*069608576	.5.2040532768	.4.6912115152	.8.93311982	.655481987
0.9	.2036600707	.3619305288	.5398824095	*0688258766	.4.108508271	.3.095108468	.7.5547677525	.761475457
1.0	.1968574952	.3181363588	.5328004735	*0679782266	.3.2169380436	.2.0627253834	.4.4454291563	.4539841046
1.1	.1910819386	.278025801	.5252237434	*0670562836	.2.6934468798	.1.3055901711	.5.5457842644	.3378740272
1.2	.1861879739	.241627839	.5171981496	*0660522318	.2.3527934636	.1.7391785		

TABLE I. ESTIMATING FUNCTIONS FOR DOUBLY TRUNCATED NORMAL SAMPLES (Continued)

ξ_2	$Z_1(\xi_1, \xi_2)$	$Z_2(\xi_1, \xi_2)$	$H_1(\xi_1, \xi_2)$	$H_2(\xi_1, \xi_2)$	$\mu_{11}(\xi_1, \xi_2)$	$\mu_{12}(\xi_1, \xi_2)$	$\mu_{22}(\xi_1, \xi_2)$	$\rho(\hat{\mu}, \hat{\sigma})$
$\xi_1 = -1.3$								
-0.8	1.4894604380	2.51785646721	.5431915317	.0815315981	12761.7147126849	6104.6668252567	2931.4852749956	.9965759648
-0.7	1.48165239292	2.51510548339	.5491151485	.0809084410	5586.1283104007	2810.7656459561	1422.4212774515	.9965733626
-0.6	0.9657145629	1.8778228136	.5594310702	.0825162631	2724.801616705	1444.4420003497	772.9253334627	.9953226265
-0.5	0.8093462561	1.6627478272	.5582460356	.07951774044	1434.84397303648	403.48862766596	456.185501922	.9931310212
-0.4	.6916221215	1.4862920699	.5616778462	.0788986519	794.9154872085	474.2655313151	286.775088275	.9901709349
-0.3	.6060860998	1.3368515006	.5638365995	.0782240234	465.713058280	292.96293395648	149.6677349127	.9861795195
-0.2	.5299136889	1.2071461635	.56533341169	.0775601522	280.4094471181	187.48566332802	130.301979768	.9831819784
-0.1	.4716069039	1.0924146579	.5659935382	.0769098773	171.3645827373	123.3831583694	92.497134277	.9736103349
0.0	.4250218205	.9894413687	.5658311167	.07627334952	109.4702849229	82.991096022	67.712276831	.9679148529
0.1	.3868126674	.8960000891	.5648661274	.07656488230	70.3492048688	56.7765727540	50.683664123	.9508580267
0.2	.3551980835	.8105196747	.5631183929	.0750314689	45.8866161358	39.3346365692	38.716171038745	.9332549111
0.3	.3288524133	.7318745164	.5606111887	.0744511132	30.347215956	27.489178922	20.1117314227	.904620525
0.4	.3067706248	.6592483670	.5573606458	.0737918008	20.3521489999	19.3243133003	23.7921507031	.8772650684
0.5	.2681781563	.5920427748	.5530854592	.07187252540	14.8725821293	13.56571416100	14.6075540526	.8361837723
0.6	.2724693117	.5298139979	.5487659567	.0724862086	9.64934647241	9.49746431368	15.4703411387	.7771453226
0.7	.2591640917	.4720277643	.543681638	.0717867792	6.8919785812	6.5810007863	12.1119748338	.7033701417
0.8	.2478774143	.4150265683	.5375480219	.0710408556	5.9394737358	4.4709512190	10.5551524235	.61093486856
0.9	.2382695675	.3700051111	.5101646620	.0702195278	3.92717645436	2.95727446172	6.853236784	.5057674272
1.0	.2301657550	.3249917683	.50398783598	.0693385398	3.1778413120	1.83994674610	7.49691111126	.1697371711
1.1	.2232718219	.2833431127	.5164327384	.0683797010	2.70367979491	1.0271627893	6.4093595234	.246827512
1.2	.2174370037	.2463884044	.5084194397	.0673324202	2.4102717137	4.4322676452	5.5207345641	.1187719724
1.3	.2125109102	.2125109102	.5000000000	.0661194026	2.2347785320	1.000000000	4.7982289710	.0000000000
$\xi_1 = -1.2$								
-0.7	1.5303015314	2.6607645889	.5391098849	.0817310779	10431.4132714657	5511.5717652520	2925.7715697719	.9976518661
-0.6	1.2198848671	2.0933370150	.5442524535	.0801769266	4519.826929096	2522.3970464672	1418.457714478	.9907013458
-0.5	1.0037121745	1.8167612321	.5455013462	.0805917188	2178.3253850380	1287.6133040532	710.0672919981	.9941705383
-0.4	.84609493959	1.6464028745	.5518650810	.0799889607	1132.65717141931	710.9695634628	454.107767242	.9913344342
-0.3	.7272370962	1.4283176383	.5543549532	.079378266	622.8633777477	416.205671869	295.7321861355	.9874361111
-0.2	.6352876767	1.2792943303	.55698464293	.0787682913	357.3260164764	254.7624048367	158.7692104047	.9837643175
-0.1	.56262969663	1.1502453176	.5567697152	.0781613139	211.7858671245	161.36071907177	129.4021085513	.976717421
0.0	.5064706535	1.0364124642	.55672449410	.0775505064	128.77255984240	104.4752596781	91.4303816149	.9666079469
0.1	.4571685021	.9345377635	.5558697989	.0767601239	79.9238757867	69.65021941618	67.1577453686	.9506844363
0.2	.4183309333	.8424139931	.5542240672	.0763608231	50.4713196712	46.6705190310	50.47446497171	.9741602757
0.3	.3861722695	.7554648927	.5518082515	.0757554374	37.2804164695	31.95141948645	38.1701917837	.9046850705
0.4	.359365021	.6815173989	.5486456703	.0751365929	21.144681461785	21.191746269929	20.40595134219	.8620132017
0.5	.3368988262	.6108079890	.5494793110	.0744949162	14.0364191193	14.46719180172	7.347693194	.9174547681
0.6	.3179847736	.54566465353	.5461718178	.0739126033	9.56371636367	10.08606130767	18.49924949447	.7502122003
0.7	.3020157370	.5465645529	.5439136548	.0731050628	6.7178036357	6.7271562469	15.465734778	.6615371996
0.8	.2885058498	.4504001521	.5290525488	.0723347164	4.4959766076	4.35561476971	12.46165719419	.5662452612
0.9	.2770642178	.3769462827	.5252758242	.071998390	3.8512983737	2.6693835372	10.4856846703	.8201516711
1.0	.2673725995	.3331667746	.5155481022	.0705903373	3.17179344660	1.4484191014	8.4055913419	.2778556048
1.1	.2591689782	.2907547925	.5080061677	.0653972194	2.7661249130	.610908510	7.4674365922	.1344236721
1.2	.2522353267	.2522353267	.5000000000	.0658130583	2.5330857157	1.0000000000	6.3920879261	.0000000000
$\xi_1 = -1.1$								
-0.5	1.5719518262	2.6044244536	.5350187453	.0819112748	9.3461.3111925871	4221.9975464792	2919.40757533707	.9976702657
-0.4	1.2610717933	2.0365726682	.53397344585	.0814183648	3.369.150164699	2236.3345986491	1414.751102207	.995209471
-0.3	1.0427949959	1.7627896533	.5428490662	.0808956901	1695.9239179355	1132.3804636205	767.7246424597	.9925433971
-0.2	.8840595405	1.5476987262	.54554610179	.0803549289	668.261688772	619.6024567967	452.2830467525	.9887380976
-0.1	.7641945655	1.3717223563	.5471913435	.0798030032	669.4811504226	359.004021776	283.1851223465	.9833861491
0.0	.61334669692	1.2232514129	.5480831964	.0792449130	764.4614040698	217.1764252093	107.42827211045	.9754562772
0.1	.53902221178	.9821625169	.5473810841	.0781152667	91.5611414546	86.8956977159	91.3361236363	.9502178996
0.2	.4911075210	.8815335731	.5452651137	.0775394958	55.65637744649	56.5567425649	56.5574252770	.9973186267
0.3	.4517455108	.7908546428	.5434967628	.07695114411	34.45751434954	37.2583344175	49.8856545617	.8086562719
0.4	.41913434267	.7085447144	.5403971415	.0763427731	21.74567021910	24.62953666161	38.009414107249	.8557352748
0.5	.3919639942	.6334297713	.5363756534	.0751054758	14.0493140648	16.2302193681	21.4258627124	.7954562518
0.6	.35919040404	.56471101056	.53270472346	.07502944640	9.3699595203	10.55565439374	23.416773010	.7126100856
0.7	.3308627531	.5011717299	.52664384810	.0743058234	6.53038616753	6.6481221437	18.790312224	.6333172217
0.8	.3338840491	.4439863303	.5209987994	.0735224292	4.48233185910	4.01204295093	15.2644645291	.667823622
0.9	.3202418971	.3911443371	.51456487803	.07266921233	3.81441676611	2.1677597655	12.5502176657	.3131395218
1.0	.3087129902	.3428970978	.5075343793	.0717362346	3.23627686429	.8972370898	13.4508679197	.1524256168
1.1	.2989732134	.2989732134	.5000000000	.0707146551	2.9217645278	.0000000000	8.7497466445	.0000000000
$\xi_1 = -1.0$								
-0.5	1.6144050970	2.3489455558	.530309190823	.0820719810	6417.75500337397	4335.5705175890	2914.7341114517	.9962745514
-0.4	1.31146566176	1.9807664583	.5348627988	.08161323497	2734.3656301561	1952.3277142042	1411.5211336928	.9937512536
-0.3	1.0829661409	1.70696246061	.5371767641	.08011636764	1276.4747274616	978.5705168155	765.3078265218	.9901212691
-0.2	.9322527260	1.4920650973	.5390095359	.0806742685	661.82045646957	529.32805139086	495.7073517413	.9847221720
-0.1	.8025112936	1.3165183640	.5399221463	.0801694746	39.1462785655	302.5475785960	282.71361053729	.9770392934
0.0	.7088749052	1.1687371345	.5401177707	.0796518248	186.64997797797	185.4282696186	185.4282696186	.9657839144
0.1	.6348062149	1.041984762	.5494616798	.0791212154	14.58340684183	110.4972733926	127.4984967607	.9493701202
0.2	.5752928229	.9297160048	.5477980817	.0785751290	61.4463312643	69.4463312643	95.4463312643	.9512471746
0.3	.5268752850	.830468003	.53571642189	.0783086617	36.47486379446	43.778112177	69.3307288411	.9867665616
0.4	.4870141817	.7413365091	.5322685518	.0774159172	22.1569118727	27.7812748364	49.6093301019	.8918661646
0.5	.454143114	.660773325	.5298129213	.0767881151	13.8686838375	17.6557850902	37.5889313449	.7651224141
0.6	.4266871746	.5876377546	.5244277624	.0761141519	9.062705175913	10.7637617552	24.6746141440	.6951802339
0.7	.4013700025	.5209605995	.5192593077	.0755808949	6.24946270766	7.44365459264	23.20947858466	.52066246637

TABLE I. ESTIMATING FUNCTIONS FOR DOUBLY TRUNCATED NORMAL SAMPLES (Continued)

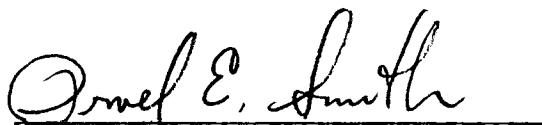
ξ_2	$Z_1(\xi_1, \xi_2)$	$Z_2(\xi_1, \xi_2)$	$H_1(\xi_1, \xi_2)$	$H_2(\xi_1, \xi_2)$	$\mu_{11}(\xi_1, \xi_2)$	$\mu_{12}(\xi_1, \xi_2)$	$\mu_{22}(\xi_1, \xi_2)$	$\rho(\hat{\mu}, \hat{\sigma})$
$\xi_1 = -0.9$								
-0.4	1.657664739	2.2942588063	.5268118752	.0822130109	4885.3889915007	3751.9116003251	2910.4016895968	.995079876
-0.3	1.3436718138	1.9259243342	.5295791326	.0818185287	2.13.5678426597	1672.1116759766	1404.7256142123	.9915353385
-0.2	1.1242397505	1.6521988381	.5314742734	.0813942734	918.7294146376	825.9704672058	764.412703957	.986200121
-0.1	.963685010	1.4376502787	.5325444029	.0809460639	449.5950113726	439.7355074366	449.3759927819	.9782974282
0.0	.8422021759	1.2627158279	.5327626089	.0804769261	231.3328985651	246.7028347409	201.7799596056	.9662742110
0.1	.7479184900	1.1715632752	.5321592148	.0799872455	123.5971404896	143.71164787946	185.731164787946	.9603077932
0.2	.6732933450	.9094815423	.5307380024	.0794749628	67.8732450537	85.5958351679	127.4767385552	.9202122172
0.3	.6133097949	.8790749693	.5285290213	.0789358056	38.2798104458	51.6471316946	90.4755783077	.8775973046
0.4	.5545034316	.7812900491	.5255487556	.0783635825	22.2317655208	31.140886195	66.0565780181	.812623782
0.5	.5244068288	.6938583090	.5218203713	.0775045659	13.4531999309	18.4528584610	49.4125815567	.71507219913
0.6	.4912160895	.6151510661	.5173700149	.0770873436	8.664563031	10.4606748681	37.7513550419	.5784C60122
0.7	.4635833567	.5440189352	.5122271384	.0763642895	6.0949773699	5.3654556720	29.3848101552	.4009212889
0.8	.4404768877	.4795547051	.5064248133	.0755705238	4.767888630	2.0967341986	23.2566776712	.1991181496
0.9	.4211010880	.4211010880	.5000000000	.0746969254	.0000000000	.0000000000	18.6859782768	.0000000000
$\xi_1 = -0.8$								
-0.3	1.7017373870	2.2403847311	.5269811119	.0823342008	3507.1593524166	3170.6490993207	2506.6916123236	.9930469592
-0.2	1.3868477994	1.8720487168	.5246651377	.0819765934	1406.5808104950	1389.4271229326	1406.3631282421	.9878813930
-0.1	1.1566210404	1.5985732935	.5257824440	.0818723633	622.011937561	674.3981861306	761.8331933965	.9796864152
0.0	1.0053686633	1.3845210996	.5260594546	.081695303	293.9177968084	350.9341230684	448.2867215017	.9667074598
0.1	.8832081711	1.2103288288	.5255081015	.0807242458	145.6025211725	191.3602834311	201.01659234073	.960248692
0.2	.7884816378	1.0643388834	.5241427546	.0802496931	74.7259197351	107.3772661682	185.1907457285	.9151466884
0.3	.7132475467	.9392492496	.5219802701	.0797419391	39.5994612433	60.9566411271	177.0904021267	.8692394833
0.4	.6531963396	.8302481604	.5190401651	.0791950566	21.8333647527	34.3297023129	90.2079243052	.7735462452
0.5	.6404185295	.7340703550	.5153447497	.0786014921	12.7986277964	18.6331761592	65.8742321535	.6417226999
0.6	.5637212563	.64849337910	.5109196181	.0779522994	8.2637071398	9.2637071398	49.2947933733	.4660470012
0.7	.5303948319	.5717041831	.5057937659	.0772376639	6.075703001C	3.4805921586	37.6833113118	.2300282120
0.8	.5026843316	.5026843316	.5000000000	.0764472927	5.1097296725	.0000000000	29.3548105811	.0000000000
$\xi_1 = -0.7$								
-0.2	1.7466148613	2.1873254672	.5185787881	.0824354095	2361.3098737174	2591.4113999112	2403.60191715178	.9896721078
-0.1	1.4309881944	1.0151432690	.5197425030	.0821626210	211.1515675735	1110.016451823	1404.4321455869	.9125915134
0.0	1.2101160788	.5460700938	.5200656930	.0822073730	385.5167086626	523.6644778543	760.5722911724	.9670774633
0.1	1.0483097634	1.3326628496	.5195858542	.0813440187	173.4491498421	262.7053035256	447.4442750124	.9430334381
0.2	.9257562509	.15953455826	.5182340758	.0809015333	81.6076504846	136.4085945430	280.4449719883	.9016806365
0.3	.8305779640	.10144702162	.5161077479	.0804379999	45.0453691477	71.4360190073	184.8051006808	.8303973246
0.4	.7552251156	.8907070345	.5131980192	.0792976221	20.8447647145	36.5007958444	126.8932871560	.7096748438
0.5	.6946173292	.7832397321	.5095263276	.0793512603	12.018800998	17.1343838928	90.0474940079	.5209873557
0.6	.6454418301	.6887890737	.5051175049	.0787208823	8.1014464477	6.2024911315	65.782185628	.2686751001
0.7	.6505080394	.6505080394	.5000000000	.0780197678	6.5394206653	.0000000000	49.255593970	.0000000000
$\xi_1 = -0.6$								
-0.1	1.7923102119	2.1350827589	.5144549060	.0825165190	1446.37994266996	2013.8284140414	2901.1320205323	.9830997827
0.0	1.4760985610	1.7672105875	.5148129559	.0822073730	526.75757680183	831.6233229895	1402.9114696187	.9673048740
0.1	1.2567301320	.1646925236	.5143394420	.0818566115	208.6570371752	373.5817999402	759.6275233995	.9383586160
0.2	1.0925160701	.1280794849	.5130457314	.0814690199	87.6383276135	174.9012724190	446.84174560087	.8830326400
0.3	.9695239149	.1097878635	.5109640572	.0810351066	39.0883544012	81.7342047573	280.0643207304	.7811802512
0.4	.8742183434	.7661606891	.5080576542	.0805513234	19.3091961767	35.6734086416	134.5747493594	.5975540458
0.5	.7986988021	.8438577233	.50404009807	.0800103545	11.4951755793	12.1547472085	126.7120423440	.3184778117
0.6	.7380491805	.7380491805	.5000000000	.0794034627	8.7457705042	.0000000000	89.9940662273	.0000000000
$\xi_1 = -0.5$								
0.0	1.8388216908	2.0836579544	.5103274728	.0825774345	761.2010635840	1437.5312202084	2899.2804553717	.9676593075
0.1	1.5221793952	1.7162524756	.5098781993	.0822797064	252.6890372503	553.9924671880	1401.8602158260	.9308041346
0.2	1.3004672866	1.4444281316	.5086063899	.0819364114	90.9939809991	223.9635052892	758.9981107662	.8522177505
0.3	1.1379931420	1.2327731455	.5065248706	.0815441677	36.2624090123	87.3800056235	446.4804872619	.6867245162
0.4	1.0193617771	1.0616515173	.5036496220	.0810975081	17.8718331494	27.2262554730	279.8741239236	.3849656602
0.5	.9194108454	.9194108454	.5000000000	.0805891546	12.4086175733	.0000000000	184.4970585327	.0000000000
$\xi_1 = -0.4$								
0.0	2.3694892136	2.5668370221	.5066304788	.0828634816	1193.0175251205	2808.9846918579	7060.1732635124	.96730477912
0.1	1.8861507533	2.0330520032	.5061975003	.0826180846	304.9026633790	662.1515802590	2898.046513173	.9117474369
0.2	1.5642341199	.66667201386	.5043993688	.0823231538	88.4495238972	276.8693630453	1401.217723112	.7604558312
0.3	1.3473306173	1.3953262145	.5028694251	.0819753556	32.2354101369	74.6235850046	758.483441410	.6711766481
0.4	1.1847446068	1.1847446068	.5000000000	.0815692415	19.1555048345	.0000000000	446.3601137719	.0000000000
$\xi_1 = -0.3$								
0.0	3.2345281600	3.3834065691	.5037386366	.0830752368	2126.2065716709	6661.5317128517	22271.9788660047	.9680370260
0.1	2.6178369486	2.5165107838	.5033155020	.0828831724	354.7482174514	1404.2047627987	7058.7272120128	.8673748821
0.2	1.9342984559	.5026660029	.0826984215	.0826984215	76.8959183311	287.13227971985	2897.4266724289	.6087121671
0.3	1.6172640800	1.6172640800	.5000000000	.0823376444	33.7364251509	.0000000000	1401.2036230246	.0000000000
$\xi_1 = -0.2$								
0.0	4.9336862098	5.0333555320	.5016644344	.0832206149	4792.7693421829	22491.8155379250	112609.2805739176	.9681533592
0.1	3.2836573156	3.3332834408	.5012462494	.0830826721	355.1029332819	2220.3397155765	22270.264864874	.7895499756
0.2	2.4668441049	2.4668441049	.5000000000	.0828897377	75.4013726417	.0000000000	7558.2452383360	.0000000000
$\xi_1 = -0.1$								
0.0	9.9667110899	10.0166869372	.5004165276	.0833054548	19192.7067056174	17998.0512569961	1800430.7131982153	.968227667
0.1	4.9833555450	4.9833555450	.5000000000	.0832222752	300.4013629333	.0000000000	112607.3521394523	.0000000000

AUXILIARY ESTIMATING FUNCTIONS FOR
DOUBLY TRUNCATED NORMAL SAMPLES

By J. David Lifsey

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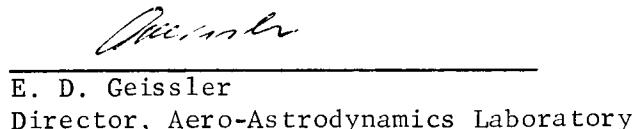
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